CS522 - Midterm Sample Problems
Computational Tools and Methods for Finance

1 Introduction

We provide a number of sample problems below. You should not consider that these problems are necessarily hints, or that there is any special relationship between the material covered below and the material that will be tested on the exam. The problems are primarily meant to illustrate the general level of difficulty that you can expect, and they offer you an opportunity to test your understanding of the material. Remember, all material covered in lectures, homeworks, handouts, and during supplementary meetings can potentially be tested.

2 Problems

1. Assume that the shape of the forward rate curve is given by $f(t) = 0.02 + 0.09\left(\frac{t}{30}\right) - 0.045\left(\frac{t}{30}\right)^2$, $0 \leq t \leq 30$ years. What is the constant, continuously compounded interest rate that you can lock in today for a future loan starting at time $t = 5$ years and maturing at time $t = 7$ years? You can assume that you can borrow and loan money for any length of time, but all such transactions must be initiated at $t = 0$.

2. Define "on-the-run" Treasuries. Explain qualitatively why the yield of these instruments is lower than that of comparable Treasuries. Does the difference between on-the-run and off-the-run instruments create any difficulties when we try to determine the shape of the underlying forward rate curve? If yes, why? If not, why?

3. If you must buy a large volume of Treasuries of a given type, would you prefer that the required Treasury be a liquid, or illiquid one? If you must sell a large volume of Treasuries of a fixed type, would you prefer that the required Treasury be a liquid, or illiquid one?

4. A strip consists of one call and two puts with the same strike price and expiration date. A strap consists of two calls and one put with the same strike price and expiration date. Assume that all transaction costs are neglijible.

   (a) Provide the payoff diagram of a strip with strike K. Provide enough points on the diagram so that the slope of all lines is unambiguous.

   (b) Provide the payoff diagram of a strap with strike K. Provide enough points on the diagram so that the slope of all lines is unambiguous.

(c) What beliefs must an investor hold to invest in a strap or a strip? When would the investor prefer to invest in a strap versus investing in a strip? When would an investor prefer a strip versus a strap?

(d) Assuming that the options that form strips and straps are European. Express the difference between the current price of a strip and a strap with the same strike K as a function of the underlying stock price.

5. Consider a one-period binomial model with parameters S, U, D, r. Consider that the money market account is denominated in dollars, i.e. $B = 1$.

   (a) Given an instrument that pays S_u in the "up" state and S_d in the "down" state, what is the time-0 price of the replicating portfolio for this payoff? What is the composition of the replicating portfolio at time $t = 0$? Comment on your observations.

   (b) Generalize the question described in the previous point for an n-period binomial model. Comment on how the answer changes, if at all.

   (c) Given an instrument that pays S_d in the "up" state and S_u in the "down" state, what is the price of the replicating portfolio for this payoff in the initial state? What if the composition of the replicating portfolio at time $t = 0$? Comment on the results.

6. Consider the binomial model built using the equivalent martingale probabilities. Assume that r, σ, and Δ are known.

   (a) Determine whether it is possible for the price of the stock in the "up" state to actually be less than the price of the stock in the initial state.

   (b) Does the answer to the question above change if you keep r fixed, and you decrease the length of the interval Δ toward 0?

   (c) Determine whether it is possible for the price of the stock in the "down" state to actually be greater than the the price of the stock in the initial state.

   (d) Does the answer to the question above change if you keep r fixed, and you decrease the length of the interval Δ toward 0?

7. Assume that we are in the context of the binomial model, and that we are using equivalent martingale probabilities. Further, assume that the underlying stock's volatility is 0. Discuss how does this situation influence the value of puts and calls.

8. Consider a two-period binomial model with $r = 6.00\%$, and volatility $\sigma = 20\%$. Assume that $\Delta = \frac{1}{2}$.

   (a) Compute the value of the corresponding parameters U, D, and q, and draw the corresponding diagram showing the evolution of the stock price.

(b) Determine the value of an American put with strike price $K = \$110$ dollars in all states of the diagram. Will there be any state in which it will be optimal to exercise the the put?

(c) Determine the composition of the replicating portfolio in the initial state, and in the two intermediate states u and d. Note that the structure of the portfolio with which you "arrive" in states u and d from the initial state is different from the structure of the portfolios that you will need to set up to replicate the payoff in the final states (assuming that you do not exercise the option earlier). Will you need additional money when you will rebalance your portfolio?

9. A chooser option is an agreement giving the holder the right to decide at a fixed time in the future T_1 whether s/he wants to obtain a European put or a call on the underlying stock. The call and the put both have the same strike price K, and they both expire at time $T_2 > T_1$. The value of the chooser option can be written as the sum of simple puts and calls written on the same underlying stock. Provide formulas that give the value of the chooser option in these terms.

   Hint 1: One possible answer is $c(T_2, K) + p(T_1, Ke^{-r(T_2-T_1)})$.

   Hint 2: This is a hard problem.

10. Consider the model of smoothest forward rate curves. Assume that the knot points are stored in array *kp*, and that their corresponding price constraints are stored in array *df*. Note that the implicit knot point $t = 0$ is not included in *kp*, nor is the implicit discount factor 1, which corresponds to knot point 0, included in *df*. You can assume that all values in *kp* are distinct, and strictly increasing, that the length of *kp* is the same as the length of *df*, and that *df*(i) corresponds to *kp*(i).

    (a) Write a Matlab function with arguments *kp* and *df* which returns the left-hand-side and the right-hand-side of the equations whose solutions give the coefficients of the smoothest forward rate curve. Assume that curves on all intervals are based at $t = 0$ (i.e. they are based at the origin of time).

    (b) Write a Matlab function with arguments *kp* and *df* which returns the left-hand-side and the right-hand-side of the equations whose solutions give the coefficients of the smoothest forward rate curve. Assume that the curve on each interval is based at the left-end of its respective interval.

11. Assume that you are given Matlab function *amput*, which values American put options on stocks that do not pay dividends. The arguments of function *amput* are, in order, the following:

    (a) r: The constant, continuously compounded interest rate earned on the money market account.

    (b) *sigma*: The volatility of the underlying stock's price.

(c) S: The initial price of the underlying stock.

(d) T: The expiration date of the put, expressed in years from the current date.

(e) K: The strike price of the put.

Write Matlab function *impvol*, which takes parameters r, S, T, K, and P (the observed market price of the put), in this order, and returns the implied volatility of the respective American put.